1. Prove that every group of order $3^2 \cdot 5 \cdot 17$ is abelian.

2. a) Prove that $\text{Aut } S_3 \cong S_3$.

   b) Prove that every nonabelian simple group $G$ is isomorphic to a subgroup of $\text{Aut } G$.

3. Let $R$ be an integral domain with 1. A non-zero, non-unit element $s \in R$ is said to be special if, for every element $a \in R$, there exist $r, q \in R$ with $a = qs + r$ and such that $r$ is either 0 or a unit of $R$.

   Prove the following:

   a) If $s \in R$ is a special element, then the principal ideal $(s)$ generated by $s$ is maximal in $R$.

   b) Every polynomial in $\mathbb{Q}[x]$ of degree 1 is special in $\mathbb{Q}[x]$.

   c) There are no special elements in $\mathbb{Z}[x]$. (Hint: Apply the definition with $a = 2$ and $a = x$)

4. Let $A_1, \cdots, A_n$ be ideals of the commutative ring $R$, and let $D = \bigcap_{i=1}^{n} A_i$. Recall that the radical $\sqrt{I}$ of an ideal $I$ of $R$ is defined as $\sqrt{I} = \{ x \in R \mid x^k \in I \text{ for some positive integer } k \}$. 

   (a) Prove that $I \subseteq \sqrt{I}$, $\sqrt{I} = \sqrt{\sqrt{I}}$ and $\sqrt{I} \subseteq \sqrt{J}$ whenever $I \subseteq J$ for ideals $I$ and $J$ in $R$.

   (b) Prove that $\sqrt{D} = \bigcap_{i=1}^{n} \sqrt{A_i}$.

   (c) Suppose that $D$ is a primary ideal and $D$ is not the intersection of elements in any proper subset of $\{A_1, \ldots, A_n\}$. Show that $\sqrt{A_i} = \sqrt{D}$ for each $i = 1, \ldots, n$.

   (Recall that an ideal $I(\neq R)$ of $R$ is primary if for any $x, y \in R$, $xy \in I$ and $x \notin I \Rightarrow y^{\ell} \in I$ for some positive integer $\ell$)