METU Mathematics Department
Graduate Preliminary Examination
Algebra I, Fall 2014

1. Let A be an abelian p-group of exponent p^m. Suppose that B is a subgroup of A of order p^m and both B and A/B are cyclic. Show that there is a subgroup C of A such that $A \cong B \oplus C$ and $B \cap C = \{0\}$.

2. Let $p > q$ be primes.

   (a) Prove that a group of order pq is not simple.

   (b) Show that there is exactly one group of order pq if $p - 1$ is not divisible by q.

   (c) Construct a nonabelian group of order pq if $p - 1$ is divisible by q.

3. Let R be a commutative ring with identity and let G be a finite group.

   (a) Show that the augmentation map from the group ring $R[G]$ to R given by the formula $f(\sum c_g g) = \sum c_g$ is a ring homomorphism.

   (b) Show that the augmentation ideal, i.e. the kernel of the augmentation homomorphism, is generated by $\{g - 1 | g \in G\}$.

   (c) If G is cyclic with generator g_0, then show that the augmentation ideal is principal with generator $g_0 - 1$.

4. Let R be a ring with identity and $f \in R[[x]]$ be a formal power series with coefficients from R.

   (a) Give a sufficient and necessary condition for f to be a unit in the ring $R[[x]]$. Prove your statement.

   (b) Classify all ideals of $\mathbb{F}[[x]]$ if $\mathbb{F}$ is a field.