1. Prove that there is no non-abelian simple group of order 36.

2. If $p$ and $q$ are primes, then show that any group of order $p^aq$ is solvable.

3. Let $F_1$ be a free group on a nonempty set $X_1$ and $F_2$ be a free group on a nonempty set $X_2$. If $|X_1| = |X_2|$, then show that $F_1 \simeq F_2$.

4. Let $R$ be a ring without identity and with no zero-divisors. Let $S$ be the ring whose additive group is $R \times \mathbb{Z}$ with multiplication defined by

\[(r_1, k_1)(r_2, k_2) = (r_1r_2 + k_2r_1 + k_1r_2, k_1k_2)\]

for any integers $k_1, k_2 \in \mathbb{Z}$ and $r_1, r_2 \in R$. Let

\[A = \{(r, 0) \mid rz + zr = 0 \text{ for all } z \in R}\].

(a) Show that $A$ is an ideal in $S$.

(b) Show that $S/A$ has an identity and contains a subring isomorphic to $R$.

(c) If $R$ is commutative, then show that $S/A$ has no zero-divisors.

5. Let $R$ be a commutative ring with identity such that not every ideal is principal.

(a) Show that there is an ideal $I$ maximal with respect to the property that $I$ is not a principal ideal.

(b) If $I$ is an ideal as in (a), show that $R/I$ is a principal ideal ring.