METU Mathematics Department
Graduate Preliminary Examination
Algebra I, February 2016

1. Prove that there is no non-abelian simple group of order 36.

2. If p and q are primes, then show that any group of order p^2q is solvable.

3. Let F_1 be a free group on a nonempty set X_1 and F_2 be a free group on a nonempty set X_2. If $|X_1| = |X_2|$, then show that $F_1 \simeq F_2$.

4. Let R be a ring without identity and with no zero-divisors. Let S be the ring whose additive group is $R \times \mathbb{Z}$ with multiplication defined by

$$(r_1, k_1)(r_2, k_2) = (r_1r_2 + k_2r_1 + k_1r_2, k_1k_2)$$

for any integers $k_1, k_2 \in \mathbb{Z}$ and $r_1, r_2 \in R$. Let

$$A = \{(r, n) \mid rx + nx = 0 \text{ for all } x \in R\}.$$

   (a) Show that A is an ideal in S.

   (b) Show that S/A has an identity and contains a subring isomorphic to R.

   (c) If R is commutative, then show that S/A has no zero-divisors.

5. Let R be a commutative ring with identity such that not every ideal is principal.

   (a) Show that there is an ideal I maximal with respect to the property that I is not a principal ideal.

   (b) If I is an ideal as in (a), show that R/I is a principal ideal ring.