

METU - Department of Mathematics
Graduate Preliminary Exam
Algebra - I
September 27, 2018

- Duration: 3 hours.

Question 1.(20 pts.) Let R be a commutative ring with identity. Show that if R contains an idempotent element e , then there exist ideals S, T of R such that $R = S \oplus T$.

Question 2.(20 pts.) Suppose that R is PID. Suppose that a, b are nonzero elements of R and they are relatively prime. Prove that $(a) \cap (b) = (ab)$.

Question 3.(20 pts.) Let $K \leq S_n$ but $K \not\leq A_n$. Then show that $[K : K \cap A_n] = 2$.

Question 4.(20 pts.) Show that no group of order 72 is simple.

Question 5.(20 pts.) Prove that subgroups of a solvable group are solvable.