Question 1. (20 pts.) Let $R$ be a commutative ring with identity. Show that if $R$ contains an idempotent element $e$, then there exist ideals $S, T$ of $R$ such that $R = S \oplus T$.

Question 2. (20 pts.) Suppose that $R$ is PID. Suppose that $a, b$ are nonzero elements of $R$ and they are relatively prime. Prove that $(a) \cap (b) = (ab)$.

Question 3. (20 pts.) Let $K \leq S_n$ but $K \not\leq A_n$. Then show that $[K : K \cap A_n] = 2$.

Question 4. (20 pts.) Show that no group of order 72 is simple.

Question 5. (20 pts.) Prove that subgroups of a solvable group are solvable.