METU-MATHEMATICS DEPARTMENT
GRADUATE PRELIMINARY EXAMINATION
ALGEBRA II, SEPTEMBER 2013

SEPTEMBER 17, 2013

1) Suppose that $f(x)$ is irreducible in $F[x]$ and K is a Galois extension of F. Show that all irreducible factors of $f(x)$ in $K[x]$ have the same degree.

2.a) Find the minimal polynomial of $i\sqrt{5}+\sqrt{2} \in \mathbb{C}$ over the rational numbers. Determine the Galois group of the splitting field of the minimal polynomial over the field of rational numbers.

2.b) Find a primitive element over the field of rational numbers for the extension field $K = \mathbb{Q}(\sqrt{5}, \sqrt[3]{4})$.

3.a) Suppose R is a commutative ring and M is an R-module. A submodule N is called pure if $rN = rM \cap N$, for all $r \in R$. Show that any direct summand of M is pure.

3.b) If M is torsion free and N is a pure submodule show that M/N is torsion free.

3.c) If M/N is torsion free show that N is pure.

4.a) Prove that any finitely generated projective module M over a PID R is free.

4.b) Is $\mathbb{Z}$-module of rational numbers $\mathbb{Q}$ projective? What about the $\mathbb{Q}$-module of rational numbers $\mathbb{Q}$?