1. Consider the entire function \( f(z) = z^2 + Bz \).
   
   a) Determine \( B \) if \( |f(B)| = |B| \) and at \( z = 0 \) the given map defines a rotation through \( \theta = \pi/4 \).
   
   b) Determine the zeros, poles and all multiple points of the holomorphic map
   \[ f : \mathbb{C} \to \mathbb{C}. \]
   
   c) Let \( g(z) = f(z)^{-n}, \ n \geq 1 \). Determine
   \[ \int_{|z|=R} \frac{g'(z)}{g(z)} \, dz \]
   for all permissible values of \( R \).

2. Consider the function \( f(z) = e^{1/(z-1)} \).
   
   a) Show that \( f \) is holomorphic in \( \mathbb{C} - \{0, 1\} \) with essential singularities at \( z = 0 \) and \( z = 1 \).
   
   b) Compute
   \[ \int_{z=R} f(z) \, dz \]
   for
   (i) \( 0 < R < 1 \), (ii) \( R > 1 \).
   
   c) True or false? Why?
   For any pair of positive real numbers \( \{r, r'\}, \ f(D^*(0; r)) \cap f(D^*(1; r')) \neq \emptyset. \)
3. Let \( \sum_{n=0}^{\infty} a_n z^n \) be a power series which converges in \( \mathbb{C} \) to a nowhere vanishing function \( f(z) \).

a) Show that for any given \( R > 0 \), there exists a positive integer \( N \) such that for all \( m \geq N \), the polynomial \( f_m(z) = \sum_{n=0}^{m} a_n z^n \) has no zeros in \( D(0; R) \).

b) True or false? Explain.
   (i) \( \log(f(z)) \) can be defined as an entire function.
   (ii) \( f \) extends to a meromorphic function on the extended complex plane if and only if it is constant.

4. a) Show that if \( f : \overline{D}(0; R) \to \mathbb{C} \) is holomorphic and \( |f(z)| < R \) on \( |z| = R \), then there exists a unique point \( a \in D(0; R) \) such that \( f(a) = a \).

b) Show that for \( a, b \in \Omega \) (\( \Omega \) a simply connected region), there exists a holomorphic automorphism
   \[ \Phi : \Omega \to \Omega \]
   such that \( \Phi(a) = b \). Is \( \Phi \) unique?