Complex Analysis Preliminary Exam
September 2018

1. (a) (10pts) Find the number of zeros of $8z^5 + 5z^2 + 2$ in the unit disk in $\mathbb{C}$.
   (b) (15 pts) Evaluate
$$\int_{|z|=1} \frac{3z^4 + 2z + 1}{8z^5 + 5z^2 + 2}$$

2. Does there exist a bounded analytic function on D which satisfies
$$f\left(\frac{1}{n}\right) = \frac{n+3}{n+2}$$
where
   (a) (10pts) $D = \{z \in \mathbb{C} : 0 < |z| < \frac{1}{3}\}$.
   (b) (15pts) $D = \{z \in \mathbb{C} : 0 < |z| < 1\}$.

3. Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the unit disk in $\mathbb{C}$.
   (a) (10pts) Find a conformal map from $\{z \in \mathbb{C} : Im\ z > -1\}$ onto $\mathbb{D}$.
   (b) (15pts) Show that if f is analytic on $\mathbb{D}$, $f(0) = 0$ and $Im\ f(z) > -1$ for all $z \in \mathbb{D}$ then $|f(z)| \leq \frac{2|z|}{1-|z|}$.

4. A function f is called doubly-periodic if for all $z \in \mathbb{C}$, $f(z) = f(z+a) = f(z+b)$ for some $a \neq b \in \mathbb{C}$.
   (a) (5pts) Show that any doubly periodic entire function must be constant.
   (b) (10pts) Let P be the parallelogram with edges $(0, a)$ and $(0, b)$. Show that if f is a doubly periodic meromorphic function in $\mathbb{C}$ which has no zeros on the boundary of P then the number of zeros of f in P is equal to the number of poles of f in P.
   (c) (10pts) Does there exist a doubly periodic meromorphic function f in $\mathbb{C}$ such that f has no poles on the boundary of P and it has only one single pole (multiplicity one) inside P ?