METU Complex Analysis Preliminary Exam
Spring 2022

1. (25 pts) Find all entire functions f satisfying $|f'(z)| \leq |f(z)|$ and $f(0) = 1$.

2. (25 pts) Find a conformal map from the unit disk $\mathbb{D}$ onto $\mathbb{C} \setminus (-\infty, 0]$.

3. (25 pts) Show that if f is an entire function satisfying

$$f(\mathbb{R}) \subset \mathbb{R} \text{ and } f(i\mathbb{R}) \subset \mathbb{R}$$

then $f(-z) = f(z)$.

4. (25 pts) Compute the integral

$$\int_0^\infty \frac{x^{-a}}{1+x} dx$$

for $0 < a < 1$.