TMS

Spring 2010

NUMERICAL ANALYSIS I

1- Consider the problem of solving a nonsingular linear system $Ax = b$.

(a) If the entries of A are given with perturbations (error) δA such that

$$(A + \delta A)(x + \delta x) = b$$

where δx is the perturbation in the solution vector x due to the perturbations in the matrix A, **prove that**

$$||\delta x|| \leq \frac{\epsilon}{1-\epsilon}||x||$$

where $\epsilon = ||\delta A||\ ||A^{-1}|| < 1$ and $||\cdot||$ is any matrix or vector norm.

(b) If the right hand side vector b is given with perturbation δb as

$$A(x + \delta x) = b + \delta b$$

where δx now, is the perturbation in the solution x due to the perturbation in b, **prove that**

$$\frac{||\delta b||}{||A||} \leq ||\delta x|| \leq ||A^{-1}||\ ||\delta b||.$$

2- Let A and B are matrices with size $n \times n$ and A is nonsingular. Consider solving the linear system

$$Az_1 + Bz_2 = b_1$$

$$Bz_1 + Az_2 = b_2$$

with $z_1, z_2, b_1, b_2 \in R^n$ and b_1, b_2 are given. **Find the condition for convergence** of the iteration method

$$\begin{aligned} Az_1^{(m+1)} &= b_1 - Bz_2^{(m)} \\ & \qquad\qquad\qquad m \geq 0 \\ Az_2^{(m+1)} &= b_2 - Bz_1^{(m)} \end{aligned}$$

where m is the iteration number.

3- Consider the two-step Newton method for

$$y_k = x_k - \frac{f(x_k)}{f'(x_k)}, \qquad x_{k+1} = y_k - \frac{f(y_k)}{f'(y_k)}$$

where k is the iteration number ($k \geq 0$). Show that, the iteration converges **cubically** to a simple root ζ of $f(x) = 0$.

4- Let

$$A = \begin{bmatrix} 6 & 2 & 1 \\ 1 & -5 & 0 \\ 2 & 1 & 4 \end{bmatrix}$$

(a) Using Gershgorin's Theorem show that the eigenvalues of A satisfies the inequality $1 \leq |\lambda| \leq 9$.

(b) Using Gershgorin's Theorem prove that all eigenvalues of a diagonally dominant matrix are non-zero.

(c) Using **power method** with $x_0 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^T$ compute the absolutely largest eigenvalue and corresponding eigenvector approximately (perform 3 iterations).