

Graduate Preliminary Examination
Numerical Analysis II
Duration: 3 Hours

1. Find the value of x for which $f(x) = 2$ by using the following table

x	0	1	2	3
$f(x)$	0	1	4	9

2. (a) Interpolate the function $\ln(x)$ by a quadratic polynomial at $x = 10, 11, 12$.
(b) Estimate the error at $x = 11.1$ when approximating $\ln(x)$ by the interpolating polynomial found in part (a).
(c) How does the sign of the error depend on x ?

3. Let $f(x) = 1 + \frac{1}{2x}$. Answer the following questions.

- (a) Apply the fixed point iteration by taking the initial point $x_0 = 1$ to compute the points x_1 and x_2 .
(b) Explain analytically if the iteration converges or diverges. Give reason.
(c) Explain graphically whether the iteration converges or diverges. (Draw a graph to show how the iteration proceeds).

4. Given that the function f has continuous second derivative on the interval $[0, 1]$. Prove that there exists a point $\xi \in (0, 1)$ such that the relation

$$\int_0^1 x f(x) dx = \frac{1}{2} f\left(\frac{2}{3}\right) + \frac{1}{72} f''(\xi)$$

is satisfied.