February, 2014

Graduate Preliminary Examination
Numerical Analysis II
Duration: 3 Hours

1. Solve the equation $f(x) = 2$ where $f(x)$ is defined by the following table

x	$f(x)$	Δ^1	Δ^2	Δ^3
0	0			
		1		
1	1		2	
		3		0
2	4		2	
		5		
3	9			

where $\Delta^1 = f(x_{i+1}) - f(x_i)$ is the forward difference of $f(x)$ at x_i.

2. (a) Show that the function

$$x_+^3 = \begin{cases} x^3 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

is a cubic spline.

(b) Show that a cubic spline on the set $\{x_i\}_{i=0}^m$ has a unique representation

$$s(x) = p(x) + \sum_{i=1}^{m-1} c_i (x - x_i)_+^3$$

where $p(x)$ is a third degree polynomial.

3. Let $p(x) = \frac{1}{2}x^2 + a_1 x + a_0$.

(a) Use

$$\int_0^1 f(x)dx = \int_0^1 f(x)p''(x)dx$$

to derive a quadrature formula for

$$\int_0^1 f(x)dx$$

which involves only the values of f and f' at the end points.

(b) Show that the formula is exact if f is a polynomial of degree≤ 1.

(c) Find a_0 and a_1 so that the quadrature rule is exact for polynomials of degree≤ 3.

4. Consider two equivalent equations

$$x \ln x - 1 = 0, \qquad \ln x - \frac{1}{x} = 0$$

in the interval $[1, 2]$.

(a) Write the Newton iteration for both formulations.

(b) By considering Newton's method as fixed point iteration find the rate of convergence of both methods. Which method is faster? The root in the interval $[1, 2]$ is $x^* = 1.7632,\ \ln(x^*) = 0.5672$.