February, 2019

Graduate Preliminary Examination
Numerical Analysis II
Duration: 3 Hours

1. (a) Show that Newton's method for the equation $f(x) = 0$ is quadratically convergent (assuming $f(x)$ is sufficiently smooth, and $f'(x) \neq 0$ at the root)

   (b) In case one can readily evaluate also $f''(x)$, the enhanced (cubically convergent) version

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{1}{2}\frac{f(x_n)^2 f''(x_n)}{f'(x_n)^3}$$

   may be used. **Derive** this formula.

2. Given the approximation formula

$$G(h) = \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}.$$

   (a) Show that

$$f'(x) - G(h) = c_1 h^3 + c_2 h^4$$

   and determine c_1 and c_2.

   (b) Find an approximation for $f'(0.35)$ using the table below and $G(h)$ given in part (a).

x	0.25	0.3	0.35	0.4	0.45
$f(x)$	0.1	0.3	0.5	0.55	0.65

   (c) Apply extrapolation to the formula $G(h)$ given in part (a) to drive an approximation to $f'(x)$ of order $O(h^4)$.

3. Consider a 3-node quadrature approximations of the form

$$\int_0^1 f(x)\,dx = af(0) + bf(1/2) + cf(2)$$

   Find a, b and c by using the following three methods:

   (a) Trapezoidal rule,

   (b) Simpson's formula,

   (c) Exact integration of the interpolating natural cubic spline.