

PRELIMINARY EXAM PROBLEMS
Differential Equations (ODE), 3 hours, 2013/2

1. Consider differential equations:

- (i) $x' = x^2$, with initial condition $x(0) = x_0 > 0$,
- (ii) $x' = x^2 + 1$, with initial condition $x(0) = x_0$.

- a) Verify that the theorem on existence and uniqueness applies.
- b) Solve for an explicit solution.
- c) What is the maximal interval of the solution?

2. Find a bounded on R solution, $x^0(t)$, of the equation $x' = -x + \sin t$. Prove that

- (a) $x^0(t)$ is a unique bounded solution of the equation;
- (b) the bounded solution is 2π -periodic;
- (c) the bounded solution is uniformly asymptotically stable.

3. Let $A(t)$ be a continuous matrix for all $t \in R$. Let $P(t)$ be the matrix solution of

$$X' = A(t)X.$$

Show that $P(t)P^{-1}(s) = P(t-s)$ for all $t, s \in R$, if and only if $A(t)$ is a constant matrix.

4. Consider the following scalar equation

$$x' = \cos x. \tag{1}$$

- (a) Find all equilibriums of the equation.
- (b) Investigate stability of the solutions by linearization.