

METU MATHEMATICS DEPARTMENT
ODE
FEBRUARY 2016 - TMS EXAM

1. Determine stable and unstable subspaces of solutions of the system

$$x' = Ax,$$

where

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & 1 & 1 \end{bmatrix}.$$

2. Consider the initial value problem,

$$x' = x^{3/4}, x(t_0) = x_0.$$

Prove that there are infinitely many solutions for any couple (t_0, x_0) , $t_0 \in \mathbb{R}$, $x_0 \geq 0$.

3. Analyze stability of all equilibriums of the pendulum equation

$$x'' + k \sin x = 0,$$

where $k > 0$ is a constant.

4. Investigate for orbital stability the solution $\xi = \sin t$ of the scalar equation

$$x'' + \mu x'(x^2 + x'^2 - 1) + x = 0,$$

where μ is a scalar parameter.