1. Determine if the following Cauchy problem has a solution in the neighbourhood of the point (1,0)

\[ yz_x - xz_y = 0, \]

a) \( z = 2y \) and \( x = 1 \)

b) \( z = 2y \) and \( x = 1 + y \).

2. Let \( \Omega = \{ x \in \mathbb{R}^3 : |x| > 1 \} \). Let \( u \in C^2(\bar{\Omega}) \) and suppose that \( u \) satisfies the Laplace equation in \( \Omega \)

\[ \Delta u(x) = 0, \text{ for all } x \in \Omega. \]

Show that

\[ \max_{\bar{\Omega}} |u| = \max_{\partial\Omega} |u| \]

if \( \lim_{x \to \infty} u(x) = 0 \).

3. For the wave equation in \( \mathbb{R}^3 \)

\[ u_{tt} = u_{xx} + u_{yy} + u_{zz} \]

find a general form of the plane wave solution. That is, a solution of the form \( u(t, x, y, z) = v(t, s) \) where \( s = \alpha x + \beta y + \gamma z \) and \( \alpha, \beta, \gamma \in \mathbb{R} \), \( \alpha^2 + \beta^2 + \gamma^2 = 1 \).

4. Consider the P.D.E.

\[ \frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} = xe^{-y}. \quad (*) \]
a) Determine the type of this PDE and find the characteristic curves.

b) Consider the given PDE as a first order PDE for \( q = \frac{\partial z}{\partial y} \).
Solve this first order PDE for \( q \). Then find the general solution of (*)

c) Find two different solutions \( z(x, y) \) which satisfy the condition
\[
z(x, x) = 1 \text{ for } x \in \mathbb{R}.
\]

d) True or false? Explain.
There exists a solution \( z(x, y) \) such that \( z(x, x) = 1, \frac{\partial z}{\partial n} = 0 \) where \( n \)
is the unit normal vector to the line \( y = x \) in \( \mathbb{R}^2 \).