PRELIMINARY EXAM PROBLEMS
Differential Equations (PDE), 2020

1. Use the Fourier method to solve the problem

$$u_t - 2u_{xx} = 0, \quad 0 < t, \quad -1/2 < x < 1/2,$$

$$u(x,0) = -x^2 + 1/4,$$

$$u(-1/2,t) = u(1/2,t) = 0, 0 < t.$$

   Hint: You may apply a shift of the variable x.

2. Prove that $3x^2y - y^3$ is a harmonic function:

   (a) use Laplace equation;

   (b) do not use Laplace equation.

3. Reduce the equation

$$yu_{xx} + xu_{yy} = 0$$

   to the canonical forms in the plane.

4. Use the method of characteristics to solve the equation

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$$

   with initial conditions:

   $(a)u(1,y,z) = y^2 + z^2$;

   $(b)u(x,y,z)|_{y^2+z^2=2} = \frac{2}{x^2}$.

5. (a) Find the general solution of the equation $u_{tt} + 9u = 0$.

   (b) Find a particular solution of the above equation with $u(x,0) = e^{-3x}\ln x, u_t(x,0) = x^2\cos x$.