PRELIMINARY EXAM PROBLEMS
Differential Equations (PDE), 2004/2

1. Solve the Cauchy problem

$$u_y = u_x^3, \quad u(x,0) = 2x^{3/2}.$$

2. (a) Verify, formally, that the PDE of the form

$$\Big\{\frac{\partial}{\partial x}\big[F(x,y)\frac{\partial}{\partial x}\big] + \frac{\partial}{\partial y}\big[G(x,y)\frac{\partial}{\partial y}\big]\Big\}\Phi(x,y) = 0$$

has a solution of the type $\Phi(x,y) = X(x)Y(y)$, if $F(x,y)$ and $G(x,y)$ are "separable" in the variables, i.e. $F(x,y) = p(x)f(y), G(x,y) = q(x)w(y)$. Then write down the system of two ODE's for $X(x)$ and $Y(y)$.
(b) If $\Phi(0,y) = \Phi(1,y) = 0$ for all y, verify that the $x-$dependence of the problem in Part (a) is equivalent to the system

$$\frac{d}{dx}\Big[p(x)\frac{dX}{dx}\Big] + \lambda q(x)X = 0, \quad X(0) = X(1) = 0,$$

where $p(x)$ and $q(x)$ are real and positive with continuous derivatives in the interval $[0,1]$ and λ is constant.

3. Use Duhamel's principle to solve the IVP

$$u_{tt} - u_{x_1x_1} - u_{x_2x_2} - u_{x_3x_3} = x_1 + x_2 + t,$$

$$u(x_1,x_2,x_3,0) = u_t(x_1,x_2,x_3,0) = 0.$$

4. Let u be a solution of IVP

$$u_t - ku_{xx} = 0, \quad x \in R, \quad t > 0,$$
$$u(x,0) = f(x).$$

where $f(x)$ is continuous on R. Assume that $u(x,t)$ tends to zero uniformly for $t > 0$ as $x \to \pm\infty$. Show that $|u(x,t)| \le M, x \in R, t > 0$, if $|f(x)| \le M, x \in R$.