METU - Department of Mathematics
Graduate Preliminary Exam

Partial Differential Equations

February, 2009

Duration: 180 min.

1. Solve the Cauchy problem

$$u_y = u_x^3, \quad u(x,0) = 2x^{3/2}$$

2. The initial value problem

$$u_{tt} - c^2 u_{xx} = x^2, \quad t > 0, \quad x \in \mathbb{R}$$

$$u(x,0) = x, \quad u_t(x,0) = 0$$

is given.

a) First state the Cauchy-Kowalewski theorem for a linear equation and then decide whether the above problem has a unique solution or not.

b) Find a particular time-independent solution of the differential equation.

c) Find the solution of the given initial value problem.

3. a) State the uniqueness theorems for the solutions of Dirichlet and Neumann problems defined for the Laplace equation $\Delta u = 0$ in $\mathbb{R}^3$.

b) Show that the solutions $u \in C^2(\Omega) \cap C^1(\overline{\Omega})$ of

$$\Delta u - [3 - (x_1^2 + x_2^2 + x_3^2)]u = 0$$

in

$$\Omega := \{x \in \mathbb{R}^3 : |x_i| < 1, \ i = 1,2,3\}$$

for the Dirichlet and Neumann problems are unique.

4. Let u be a solution of IVP

$$u_t - ku_{xx} = 0, \quad x \in R, \quad t > 0,$$
$$u(x,0) = f(x).$$

where $f(x)$ is continuous on R. Assume that $u(x,t)$ tends to zero uniformly for $t > 0$ as $x \to \pm\infty$. Show that $|u(x,t) \leq M, x \in R, t > 0$, if $|f(x)| \leq M, x \in R$.