

Problem 1.

Solve the problem

$$u_x + u_y = x \quad u(x, 0) = h(x).$$

Problem 2.For the wave equation in \mathbb{R}^3

$$u_{tt} = u_{xx} + u_{yy} + u_{zz}$$

find a general form of the plane wave solution. That is solution of the form $u(t, x, y, z) = v(t, s)$ where $s = \alpha x + \beta y + \gamma z$ and $\alpha, \beta, \gamma \in \mathbb{R}, \alpha^2 + \beta^2 + \gamma^2 = 1$.

Problem 3.

Let $\Omega = \{x \in \mathbb{R}^3 : |x| > 1\}$. Let $u \in C^2(\bar{\Omega})$ and u satisfies the Laplace equation in Ω , $\Delta u = 0 \quad x \in \Omega$. Show that

$$\max_{\bar{\Omega}} |u| = \max_{\partial\Omega} |u|$$

if $\lim_{x \rightarrow \infty} u(x) = 0$.

Problem 4.

Show that the problem

$$\begin{aligned} u_t &= -u_{xx} \quad t > 0, \quad -\infty < x < \infty \\ u(x, 0) &= f(x) \end{aligned}$$

is not well posed. Hint: Consider solutions of the equation

$$u_t = -u_{xx} \quad t > 0, \quad -\infty < x < \infty$$

$u_1(x, t) = 1$, and $u_2(x, t) = 1 + \frac{1}{n} e^{n^2 t} \sin(nx)$, $n = 1, 2, 3, \dots$