Problem 1. For the equation
\[ xz_x + yz_y = z + 1 \]
give an initial curve such that
a. there exists infinitely many solutions passing through the initial curve;
b. there exists no solution passing through the initial curve.

Problem 2. Consider a Dirichlet problem
\[
\begin{align*}
\Delta u + ku &= q(x,y) \quad \text{in} \quad x^2 + y^2 < 3 \\
u &= f(x,y) \quad \text{on} \quad x^2 + y^2 = 3
\end{align*}
\]
where \( k < 0 \). Show that a solution of the problem is unique (provided it exists).

Problem 3. Let \( u(x,y) \) satisfies
\[
\begin{align*}
e^x u_{xx} + e^y u_{yy} + xu_x + yu_y &= e^{x+y}, \quad (x,y) \in \Omega \\
u(x,y) &= -(x^2 + y^2) \quad (x,y) \in \partial \Omega
\end{align*}
\]
for some domain \( \Omega \). Show that \( u(x,y) \leq 0 \) in \( \Omega \).

Problem 4. Find a general form of a radially symmetric solution
\( u = u(r,t) \), where \( r = \sqrt{x_1^2 + x_2^2 + x_3^2} \), for the equation
\[
u_{tt} = c^2 \Delta u \quad x \in \mathbb{R}^3, \quad t \in \mathbb{R}.\]