Partial Differential Equations Feb 2018

Problem 1. For the equation

$$xz_x + yz_y = z + 1$$

give an initial curve such that
a. there exists infinitly many solutions passing through the initial curve;
b. there exists no solution passing through the initial curve.

Problem 2. Consider a Dirichlet problem

$$\begin{aligned} &\Delta u + ku = q(x,y), && \text{in} \quad x^2+y^2<3 \\ &u = f(x,y) && \text{on} \quad x^2+y^2=3 \end{aligned}$$

where $k<0$. Show that a solution of the problem is unique (provided it exists).

Problem 3. Let $u(x,y)$ satisfies

$$e^x u_{xx} + e^y u_{yy} + xu_x + yu_y = e^{x+y}, \qquad (x,y) \in \Omega$$

$$u(x,y) = -(x^2+y^2) \qquad (x,y) \in \partial\Omega$$

for some domain Ω. Show that $u(x,y) \le 0$ in Ω.

Problem 4. Find a general form of a radially symmetric solution $u = u(r,t)$, where $r = \sqrt{x_1^2+x_2^2+x_3^2}$, for the equation

$$u_{tt} = c^2 \Delta u \qquad x \in \mathbb{R}^3,\ t \in \mathbb{R}.$$