1. Prove disprove:

a) If $f : \mathbb{R} \to \mathbb{R}$ is Lebesgue integrable then the improper integral $\int_{-\infty}^{\infty} f(x) \, dm(x)$ is convergent.

b) If $\int_{-\infty}^{\infty} f(x) \, dm(x)$ is convergent then $f \in L^1$.

2. Compute $\lim_{n \to \infty} \sum_{k=0}^{\infty} \left( \frac{n}{2n+k} \right)^k$

(Hint: Use a convergence theorem)

3. Let $E \subset [0,1] \times [0,1]$ have the property that every horizontal section $E_y$ is countable and every vertical section $E_x$ has countable complement $[0,1] \setminus E_x$. Prove that $E$ is not $L$-measurable.

4. Let $(X, \sigma, \mu)$ be a measure space.

a) Define convergence in measure

b) Let $\phi : \mathbb{C} \to \mathbb{C}$ be uniformly continuous. Let $f_n, f : X \to \mathbb{C}$ be measurable and $f_n \to f$ in measure.

Show that $\phi \circ f_n$ converges to $\phi \circ f$ in measure.