METU MATHEMATICS DEPARTMENT
REAL ANALYSIS
SEPTEMBER 2013 - TMS EXAM

1. (35 pts.) Denote by χ_A the characteristic function of $A \subseteq [0,1]$

a) Prove that $\psi(t,x) := (t, \frac{x+\chi_A(t)}{2})$ is measurable if and only if A is measurable

b) Suppose A is measurable, calculate the integral $\int_{[0,1]\times[0,1]} \psi d\mu$ where μ is the Lebesgue measure on $[0,1]\times[0,1]$

c) Give an example of $A \subseteq [0,1]$ which is not Lebesgue measurable.

2. (20 pts.) Let μ be counting measure on $\mathbb{N}$. Interpret Fatou's lemma, the monotone and the dominated convergence theorems as statements about infinite series.

3. (25 pts.) a) Give an example of a continuous function $f : \mathbb{R} \to \mathbb{R}$ which maps a Lebesgue measurable set onto a non-Lebesgue measurable set.

b) Why the condinition $|f_n| \leq g \in L_1$ in the Dominated convergence theorem cannot be replaced by $|f_n(t)| \leq M \in \mathbb{R}^+$.

4. (20 pts.) Given the counting measure ν on $P(\mathbb{R})$ and the Lebesgue measure μ on the Lebesgue algebra $\sum(\mathbb{R})$.

a) Show that μ is absolutely continuous with respect to ν.

b) Explain why the Radon-Nikodym theorem is not applicable to measures ν and μ.