Real Analysis

TMS

Feb. 2016

1) Let $(X, \sum, \mu)$ be a measure space and let $f : X \to [0, \infty]$ be measurable. For $E \in \sum$ define $\nu(E) = \int_E f d\mu$ Show that ν is a measure (You will need a convergence theorem for countable additivity)

2) Let $f_n(x) = \dfrac{n^{3/2}x}{1 + n^2x^2}$ Show that $\lim_{n\to\infty} \int_0^1 f_n(x)dx = 0$

3) Let (f_n) be a sequence of integrable functions such that $f_n \to f$ a.e. with f integrable. Then prove that $\int |f_n - f| \to 0 \Leftrightarrow \int |f_n| \to \int |f|$.

4) Let h and g be integrable functions on (X, μ) and (Y, ν), and define $f(x, y) = h(x)g(y)$ Then show that f is integrable on $X \times Y$ and

$$\int_{X\times Y} fd(\mu \times \nu) = \int_X hd\mu \int_Y gd\nu$$

Hint: Take $h = X_A$, $g = X_B$ where $A \subset X, B \subset Y$ are measurable sets.