1- (4+14+7 pts) Let $X$ be a topological space and let $A$ be a subset of $X$. Let $A^o$ and $\overline{A}$ denote the interior and the closure of $A$ respectively.
(i) Give the definition of the boundary $\partial A$ of $A$.
(ii) Show that $\partial A \subset \partial A$ and $\partial A^o \subset \partial A$.
(iii) Give an example where the sets $\partial A$, $\partial A$ and $\partial A^o$ are all different.

2- (4+10+11 pts) Let $X$ be a Hausdorff space.
(i) Give the definition of a compact subset of $X$.
(ii) Show that if $A \subseteq X$ is a compact subset and if $x \in X - A$, then there are open sets $U$ and $V$ such that $A \subseteq U$ and $x \in V$ and $U \cap V = \emptyset$.
(iii) Show that if $A$ and $B$ are disjoint compact subsets of $X$, then there are open sets $P$ and $Q$ such that $A \subseteq P$ and $B \in Q$ and $P \cap Q = \emptyset$.

3- (4+10+11 pts) (i) What does it mean to say that a topological space $X$ is path-connected?
(ii) Show that $f : X \to Y$ is a continuous surjective map and if $X$ is path-connected, then $Y$ is path-connected.
(iii) Prove that $S^n = \{x = (x_0, x_1, \ldots, x_n) \in \mathbb{R}^{n+1} | x_0^2 + x_1^2 + \cdots + x_n^2 = 1\}$ as a subspace of $\mathbb{R}^{n+1}$ is path-connected.

4- (13+12 pts) Let $X$ be a topological space and let $f, g : X \to \mathbb{R}$ be continuous functions.
(i) Show that the set $A = \{x \in X | f(x) \leq g(x)\}$ is closed in $X$.
(ii) Show that the function $h : X \to \mathbb{R}$ defined by
$$h(x) = \max\{f(x), g(x)\}$$
is continuous.