Topology TMS Exam
(JUSTIFY YOUR ANSWERS)

1- (4+14+7 pts) Let X be a topological space and let A be a subset of X. Let A° and $\overline{A}$ denote the interior and the closure of A respectively.
(i) Give the definition of the boundary ∂A of A.
(ii) Show that $\partial \overline{A} \subset \partial A$ and $\partial A^\circ \subset \partial A$.
(iii) Give an example where the sets $\partial \overline{A}$, ∂A and ∂A° are all different.

2- (4+10+11 pts) Let X be a Hausdorff space.
(i) Give the definition of a compact subset of X.
(ii) Show that if $A \subseteq X$ is a compact subset and if $x \in X - A$, then there are open sets U and V such that $A \subseteq U$ and $x \in V$ and $U \cap V = \emptyset$.
(iii) Show that if A and B are disjoint compact subsets of X, then there are there are open sets P and Q such that $A \subseteq P$ and $B \in Q$ and $P \cap Q = \emptyset$.

3- (4+10+11 pts) (i) What does it mean to say that a topological space X is path–connected?
(ii) Show that $f : X \to Y$ is a continuous surjective map and if X is path–connected, then Y is path–connected.
(iii) Prove that $S^n = \{x = (x_0, x_1, \ldots, x_n) \in \mathbb{R}^{n+1} \mid x_0^2 + x_1^2 + \cdots x_n^2 = 1\}$ as a subspace of $\mathbb{R}^{n+1}$ is path–connected.

4- (13+12 pts) Let X be a topological space and let $f, g : X \to \mathbb{R}$ be continuous functions.
(i) Show that the set $A = \{x \in X \mid f(x) \leq g(x)\}$ is closed in X.
(ii) Show that the function $h : X \to \mathbb{R}$ defined by

$$h(x) = \max\{f(x), g(x)\}$$

is continuous.