GRADUATE PRELIMINARY EXAMINATION
GENERAL TOPOLOGY
Duration: 3 hours

Fall 2009

1. a) Let A be a subspace of X and U is an open subset of X. Show that if $A \cap U$ is closed in U then $U \cap A = U \cap \overline{A}$ where $\overline{A} = C\ell_X A$.

   b) Suppose that a subspace A of a topological space has the property that each of its points has a neighborhood U such that $A \cap U$ is closed in U. Show that A is open in $\overline{A}$, where $\overline{A}$ is closure of A in X as in part(a).

   c) Show that if A has the property given in part(b), then A can be written as a the intersection of an open and a closed set.

2. Let X be a compact connected Hausdoff space and $f : X \to X$ *a* continuous open map. Show that f is onto.

3. Let X be a space which is not Lindelöf. Adjoin a point p to X ~~to obtain a new space $\tilde{X}$~~ whose ~~the~~ neighborhoods of p are the sets of the form $\{p\} \cup E$ where E an open subset of X whose complement is Lindelöf. Call this new space $\tilde{X}$.

   a) Show that X is a dense subspace of $\tilde{X}$.

   b) Show that $\tilde{X}$ is Lindelöf.

4. Let $X = \{x | x : \mathbb{N} \to \mathbb{R}\}$ with the box topology (the topology which has a base $B = \{\Pi_{i \in \mathbb{N}} O_i | O_i$ open in $\mathbb{R}$ for $i \in \mathbb{N}\}$.

   a) Show that $X \times X \to X$ given by $(x, y) \to x + y$ is continuous.

   b) Show that $\mathbb{R} \times \mathbb{X} \to \mathbb{X}$ given by $(r, x) \to rx$ is discontinuous.

   c) Describe the path component of $O = (0)$.