TMS

Fall 2010

Topology

1. Let $\mathcal{B}$ be the collection of all open intervals (a, b) for $a \cdot b \geq 0$, and all differences $(a, b) - A$ for $a \cdot b < 0$ where $A = \{1/n | n = 1, 2, \cdots\}$. Show that $\mathcal{B}$ is a basis for a topology on $\mathbb{R}$. Determine $\overline{A}$ in this topology.

2. A topological space is said to be symmetric if $x \in \overline{\{y\}}$ happens if and only if $y \in \overline{\{x\}}$, where $\overline{A}$ denotes the closure of A in X for any subset A of X.

   a) Give an example of a space which is symmetric but not Hausdorff.

   b) Consider the topology on $\mathbb{R}$ in which open sets are open intervals of the form (a, ∞) for $a \in \mathbb{R}$. Show that this topology is normal but not regular. (In this question the definition of normality and regularity of a space does not assume the space is T_1).

   c) Prove that a symmetric normal space is regular.

3. Let $A = \{1, 2, 3\}$ be equipped with the discrete metric d, (i.e. $d(x, y) = 1$ if $x \neq y$, 0 otherwise), and let the Cartesian product $E = [0, 1] \times A$ be equipped with the d_∞-metric i.e.

$$d_\infty((x, a), (x', a')) = \max\{|x - x'|, d(a, a')\}.$$

   Give examples of **compact**, **non-compact**, **connected** and **disconnected** subsets of E and justify your claims.

4. All the spaces in this questions are subspaces of the real line equipped with the standard topology.

   a) Use connectedness to show that the any two intervals of the form $[a, b)$ and (c, d) are not homeomorphic.

   b) Show that any bijection $f : [0, 1) \to (0, 1)$ must have infinitely many points of discontinuity.