Topology

TMS EXAM

September 19, 2014

Duration: 3 hrs.

1. Let $\mathbb{Q}$ denote the set of rational numbers considered as a subspace of $\mathbb{R}$, the space of real numbers.

- **(a)** Show that one-point subsets are not open in $\mathbb{Q}$.
- **(b)** Show that $\mathbb{Q}$ is totally disconnected (i.e. the only connected subsets are one-point sets $\{q\}$, $q \in \mathbb{Q}$).
- **(c)** Prove that $\mathbb{Q}$ is not locally connected.

2. Let $\{X_\alpha | \alpha \in \Lambda\}$ be a family of spaces and let $A_\alpha \subset X_\alpha$ for each $\alpha \in \Lambda$.

- **(a)** Show that if A_α is closed in X_α, then $\prod A_\alpha$ is closed in $\prod X_\alpha$. Why does this imply that $\overline{\prod A_\alpha} \subset \prod \overline{A_\alpha}$?
- **(b)** Show that $\prod \overline{A_\alpha} \subset \overline{\prod A_\alpha}$

3.

- **(a)** State Urysohn Lemma.
- **(b)** Show that a connected normal space having more than one point is uncountable.

4. Let X be a compact Hausdorff space and $A \subset X$ be a closed subset.

- **(a)** Show that $X \setminus A$ is a locally compact Hausdorff space.
- **(b)** Show that the one-point compactification $(X \setminus A)^*$ of the space $X \setminus A$ is homeomorphic to the quotient space X/A. (Recall that X/A is the quotient space $X/\!\sim$ where the equivalence classes are $\{A\}$ and the single point sets.)