TMS

Spring 2010

TOPOLOGY

I. Show that for any map f from a topological space E into a topological space F, the following are equivalent:

a) f is continuous on E.

b) f^{-1} (Int A) $\subset$ Int $f^{-1}(A)$ holds for every set $A \subset F$.

c) $\overline{f^{-1}(A)} \subset f^{-1}(\overline{A})$ holds for every set $A \subset F$.

II. Let E be the set of all ordered pairs (m, n) of non-negative integers. Topologize E as follows:

For a point $(m, n) \neq (0, 0)$ any set containing (m, n) is a neighborhood of (m, n).

A set U containing $(0, 0)$ is a neighborhood of $(0, 0)$ if and only if for all except a finite number of m's the set $\{n : (m, n) \notin U\}$ is finite.

a) Show that E is not locally compact.

b) Show that E is normal.

III. Let X be a topological space and A, B be subsets of X. Show that:

a) If $(A \cap \overline{B}) \cup (\overline{A} \cap B) = \phi$ and if C is a connected set which is contained in $A \cup B$ then either $C \subset A$ or $C \subset B$.

b) If $(A \cap \overline{B}) \cup (\overline{A} \cap B) \neq \phi$ and if A and B are connected then $A \cup B$ is also connected.

IV. Let f be a continuous one-to-one map from a compact space E into a Hausdorff space F.

a) Show that $f^{-1} : f(E) \to (E)$ is continuous

b) If $A \subset f(E)$, then prove $\overline{A} \subset f(\overline{f^{-1}(A)})$.

c) Let (x_n) be a sequence of real numbers. If $\lim(2x_n + \sin x_n) = \frac{\pi}{3} + \frac{1}{2}$ then prove that $\lim x_n = \pi/6$