1. Let $T = \{ U \cup U' \mid U \subset \mathbb{R} \}$ is an open set in standard topology of $\mathbb{R}$. Show that $T$ is a topology on $\mathbb{R}$ and find the interiors and the closures of the following intervals of $\mathbb{R}$ in this topology: $A = (-2, 3)$, $B = (-\infty, 3)$, $C = (1, 3)$.

2. Let $X$ be a topological space. Prove the given statement or give a counter example to show that it is not always true.

   a) If $A \subset X$ is connected then $\overline{A}$ is also connected.

   b) If $A \subset X$ is path connected then $\overline{A}$ is also path connected.

3. Consider the space $X = \{a, b, c\}$ with the topology $\{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$.

   a) Show that $Y = \{1, 2\}$ with topology $\{\emptyset, Y, \{1\}, \{2\}\}$ is not a quotient space of $X$.

   b) Show that $Y = \{1, 2\}$ with topology $\{\emptyset, Y, \{1\}\}$ is a quotient space of $X$.

   c) Consider the function $f : X \to Y$, $f(a) = 1$, $f(b) = 1$ and $f(c) = 2$, where $Y$ has the indiscrete topology $\{\emptyset, Y\}$. Is $f$ a quotient function? Is $Y$ a quotient space of $X$?

4. Let $X$ be a countable set (you may take, for example, $X = \mathbb{Z}_+ \cap \mathbb{R}$) equipped with the discrete topology. Find a homeomorphism $f : X^+ \to A$, where $A$ is the subspace of $\mathbb{R}$ given by

   $$A = \left\{ \frac{1}{n} \mid n = 1, 2, \ldots \right\} \cup \{0\}$$

   and $X^+$ is the one point compactification of $X$. 