1. A topological space is *extremally disconnected* if and only if the closure of every open set is open. Show that for any topological space $X$ the following are equivalent:

   (a) $X$ is extremally disconnected,
   (b) Every two disjoint open sets in $X$ have disjoint closures.

2. Show the following:

   (a) An open subset of a separable space is separable.
   (b) The product of countable number of separable spaces is separable.
   (c) The quotient space of a separable space is separable.

3. Let $X, Y$ be topological spaces and $f : X \to Y$ be a continuous map. Consider the graph $G = \{(x, f(x)) : x \in X\}$ of $f$ with the subspace topology of $X \times Y$.

   (a) Show that $G$ is homeomorphic to $X$.
   (b) Show that $G$ is closed if $Y$ is Hausdorff.

4. Let $X$ be a compact Hausdorff space and $f : X \to Y$ be a quotient map. Show that the following are equivalent:

   (a) $Y$ is a Hausdorff space,
   (b) $f$ is a closed map,
   (c) The set $\{(x_1, x_2) \in X \times X : f(x_1) = f(x_2)\}$ is closed in $X \times X$. 