Topology

TMS EXAM

February 18, 2014

Duration: 3 hours

1. A topological space is *extremally disconnected* if and only if the closure of every open set is open. Show that for any topological space X the following are equivalent:

(**a**) X is extremally disconnected,

(**b**) Every two disjoint open sets in X have disjoint closures.

2. Show the following:

(**a**) An open subset of a separable space is separable.

(**b**) The product of countable number of separable spaces is separable.

(**c**) The quotient space of a separable space is separable.

3. Let X, Y be topological spaces and $f : X \to Y$ be a continuous map. Consider the graph $G = \{(x, f(x)) : x \in X\}$ of f with the subspace topology of $X \times Y$.

(**a**) Show that G is homeomorphic to X .

(**b**) Show that G is closed if Y is Hausdorff.

4. Let X be a compact Hausdorff space and $f : X \to Y$ be a quotient map. Show that the following are equivalent:

(**a**) Y is an Hausdorff space,

(**b**) f is a closed map,

(**c**) The set $\{(x_1, x_2) \in X \times X : f(x_1) = f(x_2)\}$ is closed in $X \times X$.