

Name:

Topology Feb 2018

1. Consider the set  $\mathbb{R}$  of real numbers with the topology  $\tau = \{(a, \infty) : a \in \mathbb{R}\} \cup \{\emptyset, \mathbb{R}\}$ .
  - (a) Determine whether  $(-\infty, 0)$  and  $(0, 3)$  are compact.
  - (b) Find the closure and the interior of the set  $A = [-1, 1]$ .
  - (c) Let  $x_n = 2$  for all  $n = 1, 2, 3, \dots$ . Find all limits of the sequence  $(x_n)$ .
2. Prove or disprove the followings:
  - (a) Every quotient of a Hausdorff space is Hausdorff.
  - (b) Every quotient of a path-connected space is path-connected.
3. Let  $X$  and  $Y$  be two topological spaces,  $x_0 \in X$  and let  $W$  be a (open) neighborhood of  $x_0 \times Y$  in  $X \times Y$ .
  - (a) Prove that if  $Y$  is compact, then there is a neighborhood  $N$  of  $x_0$  in  $X$  such that  $N \times Y \subset W$ .
  - (b) Show that the conclusion of (a) may not hold if  $Y$  is not compact.
4. Let  $X$  and  $Y$  be two topological spaces and  $f, g : X \rightarrow Y$  be continuous functions. Suppose that  $Y$  is Hausdorff and that there exists a dense subset  $D$  of  $X$  such that  $f(d) = g(d)$  for all  $d \in D$ . Prove that  $f(x) = g(x)$  for all  $x \in X$ .