1- (13+12 pts) Let $\mathbb{R}$ be given with the finite complement topology.
   (i) Find all limits of the sequence $x_n = n$.
   (ii) Find the interior and the closure of the set $A = (0, \infty)$.

2- (15+10 pts) Let $A, B$ be dense subsets of a topological space $X$.
   (i) Show that if $A$ is open, then $A \cap B$ is also dense.
   (ii) Give an example to show that $A \cap B$ may not be dense if $A$ and $B$ are not open.

3- (5+10+10 pts)
   (i) What does it mean to say that a topological space $X$ is compact?
   (ii) Let $f : X \to Y$ be a continuous surjective map of topological spaces, where $X$ is compact. Prove that $Y$ is compact.
   (iii) Let $Z$ be a closed subspace of a compact space $X$. Prove that $Z$ is compact.

4- (5+5+15 pts)
   (i) What does it mean to say that a topological space $X$ is connected?
   (ii) What does it mean to say that a function $p : X \to Y$ between topological spaces is a quotient map?
   (iii) Let $p : X \to Y$ be a quotient map. Prove that if each set $p^{-1} \{ \{ y \} \}$ is connected and if $Y$ is connected, then $X$ is connected.