Topology TMS Exam
(JUSTIFY YOUR ANSWERS)

1- (13+12 pts) Let $\mathbb{R}$ be given with the finite complement topology.
(i) Find all limits of the sequence $x_n = n$.
(ii) Find the interior and the closure of the set $A = (0, \infty)$.

2- (15+10 pts) Let A, B be dense subsets of a topological space X.
(i) Show that if A is open, then $A \cap B$ is also dense.
(ii) Give an example to show that $A \cap B$ may not be dense if A and B are not open.

3- (5+10+10 pts)
(i) What does it mean to say that a topological space X is compact?
(ii) Let $f : X \to Y$ be a continuous surjective map of topological spaces, where X is compact. Prove that Y is compact.
(iii) Let Z be a closed subspace of a compact space X. Prove that Z is compact.

4- (5+5+15 pts)
(i) What does it mean to say that a topological space X is connected?
(ii) What does it mean to say that a function $p : X \to Y$ between topological spaces is a quotient map?
(iii) Let $p : X \to Y$ be a quotient map. Prove that if each set $p^{-1}(\{y\})$ is connected and if Y is connected, then X is connected.