16 February 2004

Graduate Preliminary Examination
Complex Analysis
Duration: 3 hours

1. Calculate $\int_\gamma \frac{w^3+1}{w(w^4+w^3+2w^2+1)}dw$, where C is the circle of radius $1/3$ centered at 0, traced once in the clockwise direction.

2.Let $\mathbb{D} = \{z : |z| < 1\}$. Prove that there is **no** holomorphic function $f : \mathbb{D} \to \mathbb{D}$ satisfying $f(\frac{i}{2}) = 0$, $f(\frac{1}{3}) = 0$, and $f(0) = \frac{1}{5}$.

3. Suppose f is holomorphic on $\{z : |z| < 1\} = \mathbb{D}$. Prove that there is a sequence $\{z_n\}$ in $\mathbb{D}$ such that $|z_n| \to 1$ and $\{f(z_n)\}$ is bounded.
(Hint: Consider the zeroes of f.)

4. Show that $\prod_{n=0}^{\infty}(1+z^{(2n)}) = \frac{1}{1-z}$ for each $z \in \mathbb{D} = \{z : |z| < 1\}$. Prove that convergence is uniform on compact subsets of $\mathbb{D}$, but not uniform on $\mathbb{D}$.