PRELIMINARY EXAM PROBLEMS
Differential Equations (PDE), 2018/2, 3 hours

1. Consider the equation

$$xu_{xx} - yu_{yy} + \frac{1}{2}(u_x - u_y) = 0, x > 0, y > 0.$$

   (a). Reduce the equaion to the canonical form.

   (b). Find the general solution of the equation.

2. Solve the Neumann problem

$$\begin{array}{ll} \Delta u = 0, & x^2 + y^2 < 16 \\ \frac{\partial u}{\partial n} = y, & x^2 + y^2 = 16 \end{array}.$$

3. Consider the following initial value problem $u_t + uu_x = 0, u(x, 0) = g(x)$. Solve the problem by the method of characterisitics.

4. Find a solution of the problem $u_{xx} + u_{yy} = 0, -\infty < x < 0, 0 \leq y \leq h < \infty$, with $u(x, 0) = u(x, h) = 0$, $u(0, y) = 1$, and $u(x, y) \to 0$ as $x \to -\infty$.