TMS. Differential Equations (PDE)

1. (a) Find the general solution of the equation

$$xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = -xy. \tag{1}$$

   (b) Determine the solution of (1) passing through the curve $y = x^2, z = x^3$.

2. Solve the Neumann problem

$$\Delta u = 0, x^2 + y^2 < 16$$
$$\frac{\partial u}{\partial n} = y, x^2 + y^2 = 16. \tag{2}$$

3. Solve the following transport equation $u_x + yu_y + \frac{1}{x}u = 0$ if $u(1, y) = e^y$.

4. Decompose the following boundary value problem,

$$u_{tt} = c^2 u_{xx} + f(t, x),$$
$$u(0, x) = g(x), u_t(0, x) = n(x),$$
$$u(t, 0) = m_1(t), \ u(t, 5) = m_2(t), \tag{3}$$

   into three systems more suitable for solution.