Real Analysis Preliminary Exam
September 2025

1. (12+13 pts) If the following statement is true then prove it. Otherwise, give a counterexample. Show your work with details. m refers the Lebesgue measure.

   (a) Let $p \geq 1$. $L^{p+1}(\mathbb{R}, m) \subset L^p(\mathbb{R}, m)$.

   (b) Let $p \geq 1$. $L^{p+1}([0,1], m) \subset L^p([0,1], m)$.

2. (25 pts) Give an example of a compact set in $\mathbb{R}$ with strictly positive Lebesgue measure and empty interior. Show that your example works.

3. (25pts) Let $f \in L^+$ and $\int f < \infty$. Show that $\{x : f(x) > 0\}$ is σ-finite.

4. (25pts) Let $X = Y = [0,1]$, $\mu =$ Lebesgue measure and ν=counting measure, $D = \{(x,x) : x \in [0,1]\}$ be the diagonal in $X \times Y$. Compute the iterated integrals $\iint\limits_{X\times Y} \chi_D d\mu d\nu$ and $\iint\limits_{X\times Y} \chi_D d\nu d\mu$, where χ_D is the characteristic function of D.