METU MATHEMATICS DEPARTMENT

REAL ANALYSIS

FEBRUARY 2013 - TMS EXAM

Signature:

Last Name:

Name:

1.) Let (X, S, μ) be a measure space, T be a metric space. Let $f : X \times T \to \mathbb{R}$ be a function. Assume that $f(\cdot, t)$ is measurable for each $t \in T$ and $f(x, \cdot)$ is continuous for each $x \in X$. Prove that if there exists an integrable function g such that for each $t \in T$, $|f(t, x)| \leq g(x)$ for $a \cdot a \cdot x$, then $F : T \to \mathbb{R}, F(t) = \int f(x, t) d\mu(x)$ is continuous.

2.) Let $f : \mathbb{R} \to \mathbb{R}$ be measurable and positive. Consider the set of all points in the upper half-plane being below the graph of $f : A_f = \{(x, y) \in \mathbb{R}^2 : 0 \leq y < f(x)\}$

Show that A_f is $\lambda \times \lambda$ - measurable and $(\lambda \times \lambda)(A) = \int f(x) dx$.

3.) For a function $f \in L_1(\mu) \cap L_2(\mu)$ establish the following properties:

a) $f \in L_p(\mu)$ for each $1 \leq p \leq 2$; and

b) $\lim_{p \to 1^+} \| f \|_p = \| f \|_1$

4.) If $\{f_n\}$ is a norm bounded sequence of $L_2(\mu)$ then show that $\dfrac{f_n}{n} \to 0$ a.e. holds.