METU Complex Analysis Preliminary Exam
September 2023

1. (10+15 pts)

   (a) Find a conformal map from $D = \{z \in \mathbb{C} : Re\, z < 1\}$ onto the unit disc $\mathbb{D} = \{z : |z| < 1\}$.

   (b) Let f be a holomorphic function on the unit disc $\mathbb{D} = \{z : |z| < 1\}$ such that $f(0) = 0$ and $Re\, f(z) < 1$. Show that $|f(z)| \leq \frac{2|z|}{1-|z|}$.

2. (20 pts) If $a > 1$ show that the equation $z + e^{-z} = a$ has exactly one solution with positive real part.

3. (25 pts) Compute $\displaystyle\int_{|z|=1} z^n e^{1/z} dz$ where n is an integer.

4. (10+10+10 pts) Decide whether the following statements are true or false. Justify your answer!

   (a) $\mathbb{C}$ is conformally equivalent to the unit disc $\mathbb{D} = \{z : |z| < 1\}$.

   (b) $D = \{z : 1 < |z| < 2\} \setminus (1, 2)$ is conformally equivalent to the upper half plane.

   (c) $\frac{1}{z}$ has an antiderivative in $A = \{z : 1 < |z| < 2\}$.