METU Complex Analysis Preliminary Exam
September 2024

$\mathbb{D} = \{z : |z| < 1\}$ denotes the unit disc in all problems here.

1. (25 pts) Find a conformal map from $\mathbb{D} \setminus (-1, 0]$ onto $\mathbb{D}$.

2. (25 pts) Let $f : \mathbb{D} \setminus \{0\} \to \mathbb{D} \setminus \{0\}$ be a conformal map. Prove that $f(z) = e^{i\theta} z$ for some $\theta \in \mathbb{R}$.

3. (25 pts) Compute $\displaystyle\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$.

4. (25 pts) Show that if m and n are positive integers then

$$p(z) = 1 + z + \frac{z^2}{2!} + \cdots + \frac{z^m}{m!} + 3z^n$$

has exactly n zeros in $\mathbb{D}$.