METU Complex Analysis Preliminary Exam
February 2025

$\mathbb{D} = \{z : |z| < 1\}$ denotes the unit disc in all problems here. Show your work.

1. (15+10 pts)

   a) Find a conformal map from $\{z = x + iy \in \mathbb{C} : x + 1 < y < x + 2\}$ onto $\mathbb{D}$.
   b) Is there any conformal map from $\mathbb{C} \setminus \{0\}$ onto $\mathbb{D} \setminus \{0\}$? Justify your answer.

2. (25 pts) Compute $\displaystyle\int_0^\infty \frac{\cos x}{(1+x^2)^2} dx$.

3. (25 pts) Find the closure of the image $f(\mathbb{D} \setminus \{0\})$ where $f(z) = e^{1/z}$. Hint: What type of singularity does f have at $z = 0$?

4. (25 pts) Find the number of zeros of $f(z) = z^7+4z^4+z^3+1$ in the annulus $\{1 < |z| < 2\}$.