February, 2012

Graduate Preliminary Examination
Numerical Analysis II
Duration: 3 Hours

1. The following integration formula is Gaussian quadrature type

$$\int_{-1}^{1} f(x)\,dx = f(-\frac{1}{\sqrt{3}}) + f(\frac{1}{\sqrt{3}})$$

   (a) Derive this formula.

   (b) Determine a formula for the integration

$$\int_a^b f(t)\,dt$$

   (c) By using part (a) and (b), evaluate

$$\int_0^{\pi/2} t\,dt$$

2. Assume that f be a 3 times continuously differentiable function near a root α. Show that the iterative process

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{[f(x_n)]^2 f''(x_n)}{2[f'(x_n)]^3}$$

is a third order process for solving $f(x) = 0$.

3. Estimate the multiple integral

$$I = \int_0^1 \int_1^{e^x} \left(x + \frac{1}{y}\right) dydx$$

numerically by using

   (a) Trapezoidal rule in both x and y directions.

   (b) *Composite Trapezoidal* rule in x direction and Trapezoidal rule in y direction.

4. By using Newton form of an interpolating polynomial show that

(a) If $p(x) \in \mathcal{P}_n$ (the set of all n-th degree polynomials) interpolates a function f at a set of $n+1$ distinct nodes $x_0, x_1, \ldots x_n$ and if t is a point different from the nodes, then

$$f(t) - p(t) = f[x_0, x_1, \ldots, x_n, t] \prod_{j=0}^{n} (t - x_j).$$

(b) If $f \in C^n[a, b]$ and if $x_0, x_1 \ldots, x_n$ are distinct points in $[a, b]$ then there exists a point $\eta \in (a, b)$ such that

$$f[x_0, x_1, \ldots, x_n] = \frac{f^{(n)}(\eta)}{n!}.$$

(c) If f is a polynomial of degree k, then for $n > k$,

$$f[x_0, x_1, \ldots, x_n] = 0.$$