PRELIMINARY EXAM PROBLEMS
Differential Equations (ODE), 3 hours, 12.09.2006

1. Consider the differential equation

$$y'' + q(x)y = 0, \tag{1}$$

where $q : [\alpha, \beta] \to R$ is a continuous function such that $0 < m \leq q(x) \leq M$. Let $\{x_1, x_2, \ldots, x_n\}$ be the zeros of a solution $y(x)$ such that $\alpha \leq x_1 < x_2, < \ldots < x_n \leq \beta$.

Show that:

(*a*) $\frac{\pi}{\sqrt{M}} \leq x_{i+1} - x_i \leq \frac{\pi}{\sqrt{m}}, i = 1, 2, \ldots, n-1;$

(*b*) $\frac{\sqrt{m}}{\pi}(\beta - \alpha) < n + 1.$

2. Applying the differentiable dependence of solutions on the initial value estimate the deviation of a solution $y(t) = y(x, 0, y_0)$ of the equation $y' = y + \sin y$ on $[0, 1]$ if the initial value is changed from 0 to y_0 and $|y_0| < 0.01$.

3. (*a*) Find all values of a parameter $a \in R$ such that the system

$$x' = 2y - 4x + 1, \quad y' = 2x - y + a$$

has solutions bounded on R.

(*b*) Define all these bounded solutions.

(*c*) Are these solutions stable?

4. For the initial value problem

$$y' = \lambda + \cos y, y(0) = 0,$$

find an upper estimate for $|y(x, \lambda_1) - y(x, \lambda_2)|$ and deduce that $y(x, \lambda)$ is continuous.