TMS. Differential Equations (ODE)

1. Find a solution $y(x) = \phi(x)$ of the equation $x^2 \frac{dy}{dx} - 1 = \cos(2y)$ such that $\lim_{x\to\infty} \phi(x) = \frac{5\pi}{4}$.

2. Investigate stability of the system $x' = -4x + 6y, y' = -3x + 5y$. Draw the phase portrait.

3. Find a $2\pi-$periodic solution for the equation $y' = 2y\sin^2(x) + \cos(x)$. Investigate stability of the solution.

4. Find a solution $(x, y) = (\phi(t), \psi(t))$ of the system $x' = 2x + y - 7te^{-t} - 3, y' = -x + 2y - 1$, which is bounded as $t \to \infty$, and evaluate $\lim_{t\to\infty} \phi(x), \lim_{t\to\infty} \phi(t)$.