

**PRELIMINARY EXAM PROBLEMS**  
**Differential Equations (ODE), 3 hours, 2013/2**

1. Consider differential equations:

- (i)  $x' = x^2$ , with initial condition  $x(0) = x_0 > 0$ ,
- (ii)  $x' = x^2 + 1$ , with initial condition  $x(0) = x_0$ .

- a) Verify that the theorem on existence and uniqueness applies.
- b) Solve for an explicit solution.
- c) What is the maximal interval of the solution?

2. Find a bounded on  $R$  solution,  $x^0(t)$ , of the equation  $x' = -x + \sin t$ . Prove that

- (a)  $x^0(t)$  is a unique bounded solution of the equation;
- (b) the bounded solution is  $2\pi$ -periodic;
- (c) the bounded solution is uniformly asymptotically stable.

3. Let  $A(t)$  be a continuous matrix for all  $t \in R$ . Let  $P(t)$  be the matrix solution of

$$X' = A(t)X.$$

Show that  $P(t)P^{-1}(s) = P(t-s)$  for all  $t, s \in R$ , if and only if  $A(t)$  is a constant matrix.

4. Consider the following scalar equation

$$x' = \cos x. \tag{1}$$

- (a) Find all equilibriums of the equation.
- (b) Investigate stability of the solutions by linearization.