TMS. Differential Equations (ODE)

1. Draw integral curves (the phase portrait) of the scalar equation

$$\frac{dy}{dx} = \frac{x-y}{|x-y|}. \tag{1}$$

2. Consider the Riccati equation

$$\frac{dy}{dx} = y^2 + f(x), \tag{2}$$

where $f(x)$ is an $\omega-$periodic function. Prove that

$$\int_0^\omega (y_1(x) + y_2(x))dx = 0,$$

where y_1, y_2 are two $\omega-$periodic solutions of the equation (2).

3. Analyze Lyapunov stability of the following initial value problem,

$$\tfrac{dx}{dt} = \tfrac{a}{t}x,\ x(1) = 0,$$

where a is a real parameter.

4. Solve the equation

$$x^2\frac{d^2y}{dx^2} - 2y = 0,$$

with boundary conditions $a)\, y(1) = 1,\ \lim_{x\to\infty} y'(x) = 0, b)\ \lim_{x\to 0} y(x) = 0,\ y'(1) = 1.$