TMS. Differential Equations (ODE)

1. Compute the first four successive approximations for the solution of the equation

$$\frac{dy}{dx} = y^2, y(0) = 1. \tag{1}$$

   Compare the result with the exact solution.

2. **a)** Find the solution $x(t) = x(t, t_0, x_0)$ for the equation $x' = tx, t > 0, x > 0$. Prove continuous dependence of the solution in the initial data by using the definition of the dependence and solution of the initial value problem, without applying the theorem.

   b) Find the derivative $\frac{\partial x(t)}{\partial t_0}$ of the solution in part *a*) by: i) applying the theorem on the differentiability; ii) direct differentiate the solution in the initial data; 3) compare the results in i) and ii).

3. **a)** Show that the equation $\frac{d^2x}{dt^2} + x = f(t), x(0) = 0, x'(0) = 0$ have a solution

$$x(t) = \int_0^t \sin(t - s) f(s) ds.$$

   b) Precise the solution in part *a*), when the function $f(t)$ is piecewise such that $f(t) = 0$ for $t < 1$, and $f(t) = t$ for $t \geq 1$. Determine, if the solution is a continuous function.

4. Analyze Lyapunov stability of solution for the following initial value problem,
   $\frac{dx}{dt} = 1 + t - x$, $x(0) = 0$.