TMS - Differential Equations (ODE)

1. Find successive approximations $y_0(x), y_1(x), y_2(x), y_3(x)$ for the initial value problem $y' = -2y + 1, y(0) = 1$.

2. Analyze periodicity of solutions for the system $x_1' = x_2, x_2' = \sin(t)x_2$.

3. Differentiate the Cauchy solution $x(t) = X(t, t_0)x_0 + \int_{t_0}^{t} X(t, s)f(s)ds$ of the inhomogeneous system $x' = A(t)x + f(t)$ to find $\frac{\partial x}{\partial t_0} \frac{\partial x}{\partial x_0^j}, j = 1, 2, \ldots, n$. Moreover, use the solution to analize continuous dependence on the initial data.

4. Consider the linear differential equation $x' = a(t)x + b(t)$, where $a(t)$ and $b(t)$ are continuous real functions on the half-axis $t \geq 0$. Prove that the following statement is valid: If $a(t) \leq -m < 0$, then any solution of the linear equation is asymptotically stable. Find a sufficient condition for the function $b(t)$ such that all solutions are bounded on the positive half-axis $t \geq 0$.

5. Consider the differential equation $x' = a(t)x$, where $a(t)$ is a continuous real function on the half-axis $t \geq 0$. Prove that the zero solution of the equation is unstable, if f $a(t) \geq m > 0$.