

## Partial Differential Equations

Problem 1. For the equation

$$(y^2 - x)z_x + yz_y = z$$

- (a) give an example of the initial curve so that the corresponding Cauchy problem has a unique solution (do not solve the problem); (Explain)  
(b) give an example of the initial curve so that the corresponding Cauchy problem has no solution. (Explain)

Problem 2. In the region  $D := \{(x, y) : y > 1\}$  determine the type and transform to the canonical form the following equation

$$x^2 u_{xx} - 2xy u_{xy} + y^2 u_{yy} = 0.$$

Problem 3.

- (a) Show that if  $u(x, t)$  satisfies

$$\begin{aligned} u_{tt} + u_t &= u_{xx}, & 0 < x < L, t > 0 \\ u(0, t) &= 0, u(L, t) = 0 & t \geq 0 \end{aligned}$$

then  $E(t) = \int_0^L (u_t^2 + u_x^2) dx$  is a decreasing function.

- (b) Use (a) to prove uniqueness of solution for the initial-boundary value problem

$$\begin{aligned} u_{tt} + u_t &= u_{xx}, & 0 < x < L, t > 0 \\ u(x, 0) &= f(x), u_t(x, 0) = g(x) & 0 \leq x \leq L \\ u(0, t) &= a(t), u(L, t) = b(t), & t \geq 0 \end{aligned}$$

Problem 4. Does the following problem has analytic solution in the neighborhood of the point  $(1, 1)$

$$u_{tt} = u_{xx} u_x^2, u(x, 1) = \sin x, u_t(x, 1) = e^x ?$$

(Explain).