TMS. Differential Equations (PDE)

1. (a) Find the general solution of the equation

$$xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = -xy \tag{1}$$

   (b) Determine the solution of (1) passing through the curve $y = x^2, z = x^3$.

2. Reduce the equation $yu_{xx} + xu_{yy} = 0$ to the canonical forms in the plane.

3. Suppose $u(x,t)$ is the solution to

$$\begin{array}{ll} u_t - u_{xx} = x, & 0 < x < 1,\ t > 0 \\ u(0,t) = 0,\ u(1,t) = 0 & t \geq 0 \\ u(x,0) = 0 & 0 \leq x \leq 1. \end{array}$$

   Apply the maximum principle to show that $u(x,t) \leq \dfrac{x - x^3}{6}$ for $0 < x < 1$ and $t > 0$.

4. Find a harmonic function $u(r,\theta)$ in the annulus $2 < r < 4$ with $u(2,\theta) = 1$ and $u(4,\theta) = \sin^2 2\theta$.