TMS- PDE

Feb. 15, 2016

1) Consider the p.d.e. $x^4p^2 - yzq - z^2 = 0$, where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$, in the region $\{(x, y, z) \in \mathbf{R}^3 : x > 0, y > 0, z > 0\}$.

a) Write the p.d.e. obtained by applying the transformation

$$X = \frac{1}{x}, Y = \frac{1}{y}, Z = \ln(z)$$

to the given equation.

b) Solve the p.d.e. you obtained in (a) to get the **complete integral of the original p.d.e.**

c) Find all singular solutions, if any, of the given equation.

2) Let r, θ denote the polar coordinates in the plane. Find **all** harmonic functions $u(r, \theta)$ in the region $\mathcal{R} = \{(r, \theta) : r > 1\}$, which satisfy $u(1, \theta) = 1 + \cos(\theta)$

(a) if u is bounded in $\mathcal{R}$.

(b) if u is unbounded in $\mathcal{R}$.

3) Consider the p.d.e. $x^2\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} = 0.$

a) Determine the regions in $\mathbf{R}^2$ in which the given equation is (i) elliptic, (ii) parabolic, (iii) hyperbolic.

b) Find the normal form of this equation in the region $\{(x, y) : x > 0\}$.

c) Using the normal form, find the general solution of the given equation for $x > 0$.

4) Solve the boundary value problem

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ in $\{(x, y) \in \mathbf{R}^2 : \pi > x > 0, b \geq t > 0\}$,

$u(0, t) = u(\pi, t) = 0,\ u(x, 0) = 1 - \cos(4x).$